Abstract of the monograph

"**Solution Methods for Variational Problems of Low Thrust Spaceflight Mechanics**"

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At present, the mechanics of low thrust space flight has grown into an independent department of spacecraft dynamics, which considers the problems of trajectory and motion laws optimization, as well as the choice of optimal mass proportions of a spacecraft with electric propulsion system, as a complex. Physical and technical background of electric propulsion are well covered in literature. At present, in Russia (formerly USSR) and a number of other countries – the USA, Germany, France, Great Britain – various designs of low thrust propulsion systems have been invented, produced and used in numerous applied space flight missions. Electric propulsion systems are mostly used in near-Earth space for satellite orbit control. Electric propulsion system are supplied with electric power by a solar (and, in future, nuclear) power plant.

The experiments in outer space, performed in the last decade, demonstrated that the electric propulsion system characteristics obtained by theory-based calculations well match the experimental data, showed the directions for further improvement of their efficiency, and proved the possibility of creating, in future, the spacecraft which would rely only on electric propulsion systems.

The major direction for theoretical research in low thrust spaceflight dynamics is the development of analytical and numerical methods of solution for variational problems of optimal trajectory search. Recently the issues concerning the consideration of additional factors in mathematical models of spacecraft motion, as well as additional constraints on propulsion control, are gaining importance.

For instance, it should be noted, that most published works consider a spacecraft as a variable mass point, thus reducing the problem of optimal trajectory choice to thrust vector control optimization. However, if the thruster is rigidly fixed relative to the body of the spacecraft, any thrust vector change program can be performed only by turning of the body of the spacecraft in space with the help of controlling momentum, the magnitude of which is limited. In this case, the optimal trajectory must be chosen out of the condition of minimal total expenditure for center of mass motion control and programmed spacecraft body rotation, or allowing for the constraints of thrust vector orientation. Hence the mathematical model of optimization problem becomes significantly more sophisticated.

Solar powered spacecraft control presents additional complications. The main cause of difficulties is the presence of large solar panels with individual solar orientation systems, which imposes limitations on optimal thrust vector control. On the other hand, constraints connected with solar panel orientation management prohibit the complete utilization of the torque generated by the electric propulsion systems. For this reason, a combined optimization of not only trajectorial and angular motion, but also the solar panel orientation, and the initial position of the orbital plane relative to the Sun, is necessary.

Therefore, the traditional approach to low thrust spaceflight mechanics, wherein the spacecraft is considered as a material point with «perfect» thrust vector control, in a number of cases is apparently untenable.

Another complication is the difficulty of solving the optimization problems for multi-loop flight trajectories in «strong» gravitational fields of the planets. A brilliant approach to the solution of this problem is separation of the variables into «fast» and «slow-changing» and averaging of the equations of motion. This monograph further develops this approach.

Considerably difficult are also the problems of close-loop interplanetary transfer optimization. Separate intervals of the transfers (near-planet, heliocentric) are relatively easy to optimize, however, in order to unite them into a single close-loop transfer scheme, an interval joining problem with parametric optimization has to be solved. The algorithm of the solution of a variation problem has an obvious two-cycle nature, and its stability becomes problematic.

When solving the motion optimization problems for a solar-sail powered spacecraft, the existing interaction between the magnitude and the direction of controlling acceleration requires combined modeling of the motion of the spacecraft's center of mass, and its rotation in relation to the center of mass. Additional complications are created by the need to account for the light and shadow situation on the orbit, the degrading effect of temperature and radiation on the solar sail, as well as restrictions on control, resulting from large size and low integrity of the construction.

Authors also suggest a new approach to the search of the optimal control for low thrust spacecraft moving through the system of two gravitating bodies. Such maneuver was performed, for example, by the SMART-1 spacecraft (the problem of transfer to Moon satellite orbit) on the interval between the spheres of influence of the Moon and the Earth. Optimization of this motion within the SMART-1 project was not carried out. The difficulties with optimization of this stage of the transfer are caused by lengthy and tedious mathematical model of motion, as well as the need for optimal joining with other parts of the trajectory.

When the mathematical model of spacecraft motion grows more sophisticated, new difficulties in solution of optimization problems arise. What actually happens is the contraction of the range of allowable trajectories and controls. Under these conditions, the role of approximate methods of solutions for optimization problems is gaining importance. Therefore, development of the methods of finding near-optimal trajectories and their evaluation becomes an urgent issue. These methods are based on the well-known method of extension of the range of allowable states and controls and sufficient conditions for absolute minimum.

The authors suggest and theoretically justify the following approach to the solution of low thrust spaceflight optimization problem. At first, the problem is set in most general manner. Then the analysis focuses on physical nature of the problem and those of its characteristics which could allow the reduction of the mathematical model of the motion, for instance, averaging the cyclical motions (rotation of the spacecraft around the center of mass, multi-loop orbital motion, etc.), or neglecting certain relations and constraints. As the result, the model of the motion is considerably simplified, retaining, however, all significant characteristics of the initial model.

For finding near-optimal trajectories, the authors suggest a method of breaking the state space into separate regions, where possible simplifications of the model of motion allow to obtain analytical results for quantificational evaluations. When near-optimal solutions for every region have been obtained, they are joined on the boundaries of the regions by state coordinates - parameters of the joining points. With this approach, one has to consciously abandon the attempt to obtain a universal solution for a priori given range of allowable states.

The next step is finding the structure of optimal control for the simplified model of motion. Both the methods of optimal systems theory and heuristic methods of finding near-optimal controls are applied here. The latter have the advantage of enabling one to choose the rational controls, which assuredly satisfy the constraints and contain the number of free parameters necessary for the solution of a boundary problem.

The third stage is the estimation of the degree of optimality of the chosen near-optimal controls and trajectories. Special evaluation procedures allow to estimate how near the obtained controls are to perfectly optimal, and indicate the ways of their improvement.

Combined optimization of trajectories and design parameters of the spacecraft is suggested to be performed by method of iterations, using a sequence of increasingly sophisticated dynamic and design models. According to this approach, the solution of optimization problem within a simplified model is seen as first approximation. On consequent iterations, as the model becomes more sophisticated, the cost of the working fluid on realization maneuver, and the design parameters which define the layout of the spacecraft obtain more precision as well is made more precise.

This monograph can be useful for engineers and researchers in the area of designing spacecraft with low thrust engines and working in related fields of aerospace technology.